Dark and antidark diffraction-free beams

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We present dark and antidark diffraction-free beams and discuss their properties. We show that all such beams must be partially spatially coherent. The new beams can be used for optical trapping of atoms.

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Optical waves and pulses that are immune to diffraction and dispersion have been in the focus of attention of the optical community owing to their particle-like behavior, which may be useful for many potential applications. It is often believed that the formation of such wave packets requires a nonlinear medium. Indeed, the entire field of soliton optics is devoted to studying the possibility of balancing dispersion or diffraction with the nonlinear response of an optical medium [1].

To date, two profoundly different kinds of optical solitons have been discovered, bright and dark ones. While a bright soliton intensity profile falls off with the distance away from the soliton center, a dark soliton profile has a dark notch at the center, with the intensity increasing toward the soliton periphery and tending to a constant value, known as a cw background intensity [2]. As the antidark solitons are mirror images of the corresponding dark ones, their intensity profiles appear as bright (higher intensity) spots on a cw background. The fundamental difference between bright solitons on one hand and dark or antidark solitons on the other, is that the former carry a finite amount of energy whereas the latter do not.

Perhaps somewhat surprisingly, nonspreading wave packets can also form in free space or linear media. However, to satisfy the standard Fourier reciprocity relation [3], any nonspreading linear wave packet must contain infinite energy. Many such solutions of the paraxial and nonparaxial optical wave equations have been found to date [4–12]. To our knowledge, however, *all* nonspreading beams and pulses discussed in the literature thus far can be classified as bright. A fundamental question thus arises: Do there exist *dark or antidark* diffraction-free beams whose intensity profiles have the features similar to those of dark or antidark solitons?

In this Letter, we address this issue by demonstrating that such diffraction-free beams can indeed be generated, provided spatial coherence of the beams is less than perfect. We stress that the requirement that dark and antidark diffraction-free beams be *necessarily partially spatially coherent* is a novel and surprising feature, which has no analog in the soliton realm. We explain a physical origin of partial spatial coherence of the new beams and propose a method for their experimental realization. We also briefly compare basic features of dark and antidark diffraction-free beams with those of the corresponding solitons.

We begin by considering a partially coherent beam whose free space propagation is governed by a paraxial wave equation of the form [13]

$$(2ik_0\partial_z + \nabla_{\perp 1}^2 - \nabla_{\perp 2}^2)W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) = 0. \qquad (1)$$

Here the cross-spectral density function, $W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z) \equiv \langle U^*(\boldsymbol{\rho}_1,z)U(\boldsymbol{\rho}_2,z)\rangle$, specifies second-order correlations of the optical fields U at a pair of points with the positions, defined by the radius vectors $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ in any transverse plane $z=\text{const}>0$ of the beam; $k_0=\omega_0/c$, and the angle brackets denote the ensemble averaging [14].

General structure of partially coherent diffraction-free beams. As pointed out in [15,16], the cross-spectral density of any diffraction-free beam must be independent of z. To obtain a general form of the cross-spectral density of optical fields associated with such beams, we express Eq. (1) in the new variables $\mathbf{r}=\boldsymbol{\rho}_2-\boldsymbol{\rho}_1$ and $\mathbf{R}=(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)/2$ as

$$(ik_0\partial_z + \nabla_{\mathbf{r}}\cdot\nabla_{\mathbf{R}})W(\mathbf{r},\mathbf{R},z) = 0. \qquad (2)$$

It then easily follows from Eq. (2) that a general z-independent solution is given by the expression

$$W_{df}(\mathbf{r},\mathbf{R}) = \Phi(\mathbf{r}) + \Psi(\mathbf{R}), \qquad (3)$$

where Φ and Ψ are arbitrary functions. The cross-spectral density of Eq. (2) represents a physical beam only if it is Hermitian and nonnegative definite. The Hermiticity follows from the definition of the cross-spectral density and implies that [14]

$$W(\boldsymbol{\rho}_2,\boldsymbol{\rho}_1,z) = W^*(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,z). \qquad (4)$$

It follows at once from Eq. (3) that W_{df} is Hermitian if the independent functions Φ and Ψ satisfy the conditions

$$\Psi^*(\mathbf{R}) = \Psi(\mathbf{R}), \quad \Phi^*(\mathbf{r}) = \Phi(-\mathbf{r}). \qquad (5)$$

The nonnegative definiteness means in physical terms that the beam carries a positive or zero amount of energy. Mathematically, it can be represented by the inequality

$$\int\int \mathrm{d}\boldsymbol{\rho}_1 \mathrm{d}\boldsymbol{\rho}_2 W_{df}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) f^*(\boldsymbol{\rho}_1) f(\boldsymbol{\rho}_2) \geq 0, \tag{6}$$

which must hold for any square-integrable function f [14]. Alternatively, the cross-spectral density can be expanded in a series of coherent modes as

$$W_{df}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) = \sum_{\nu} \lambda_\nu u_\nu^*(\boldsymbol{\rho}_1) u_\nu(\boldsymbol{\rho}_2), \tag{7}$$

and the nonnegative definiteness is guaranteed by the requirement that the modal weights λ_ν, specifying the powers carried by the modes, be real and nonnegative:

$$\lambda_\nu \geq 0. \tag{8}$$

Equations (3), (5), and (6) or (8) determine *all* possible partially coherent diffraction-free beams.

Dark and antidark diffraction-free beams. Let us now focus on a particular class of partially coherent diffraction free beams whose cross-spectral density is given by the expression,

$$W_{df}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) \propto J_0(\beta|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|) + \alpha J_0(\beta|\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2|), \tag{9}$$

where α and β are arbitrary constants, β being real valued, and $J_0(x)$ is a zero-order Bessel function. The intensity of any such beam is

$$I_{df}(\boldsymbol{\rho}) \equiv W_{df}(\boldsymbol{\rho},\boldsymbol{\rho}) \propto 1 + \alpha J_0(2\beta\rho). \tag{10}$$

It readily follows from Eq. (10) that for any physical beam, $I_{df} \geq 0$, resulting in the following constraint on the values of α:

$$\alpha^* = \alpha, \quad |\alpha| \leq 1. \tag{11}$$

To show that condition (11) guarantees that Eq. (9) describes physical beams and gain further insight about the new beams, we derive a coherent mode expansion of the cross-spectral density of Eq. (9). With this purpose, we recall the so-called summation theorem for Bessel functions [17]:

$$J_0(\beta|\boldsymbol{\rho}_1 \mp \boldsymbol{\rho}_2|) = \sum_{m=-\infty}^{\infty} (\pm 1)^m e^{im(\phi_2-\phi_1)} J_m(\beta\rho_1) J_m(\beta\rho_2). \tag{12}$$

It can be inferred from Eqs. (7), (9), and (12) that

$$W_{df}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) \propto \sum_{m=-\infty}^{\infty} \lambda_m \psi_m^*(\boldsymbol{\rho}_1) \psi_m(\boldsymbol{\rho}_2), \tag{13}$$

where the coherent modes are Bessel beams such that

$$\psi_m(\rho) = J_m(\beta\rho) e^{im\phi}, \tag{14}$$

and the modal weights are found to be given by

$$\lambda_m = 1 + (-1)^m \alpha. \tag{15}$$

It can be easily concluded from Eqs. (11) and (15) that nonnegativity criterion (8) is satisfied for the novel beams.

Qualitatively, the cross-spectral density of Eq. (9) describes dark ($\alpha<0$) or antidark ($\alpha>0$) diffraction-free beams. The corresponding intensity profiles are displayed in Fig. 1 for the two values of the parameter $\alpha=1,-1$. It is seen from the figure that the value $\alpha=-1$ leads to "black" diffraction-free beams with a dark notch in the center. The analysis of Eq. (6) further reveals that in the interval $-1<\alpha<0$ we have "gray" diffraction-free beams. In this connection, it is interesting to note that the partially coherent dark solitons were shown to be necessarily gray [18].

The coherent mode expansion of the present dark and antidark diffraction-free beams hints at a possible way of generating them in the laboratory. It can be noticed that a diffraction-free beam can be obtained as an incoherent superposition of a finite number N of Bessel modes of Eq. (14) as $W_{df}^{(N)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2) \propto \sum_{m=-N}^{N} \lambda_m \psi_m^*(\boldsymbol{\rho}_1) \psi_m(\boldsymbol{\rho}_2)$, which does satisfy Eq. (1) as any Bessel mode is a solution to the equation $\nabla_\perp^2 \psi_m = -\beta^2 \psi_m$. The incoherent superposition of Bessel modes can be realized in experiment by adding up the outputs of several *independent* sources, each of which is generating a particular Bessel mode. Coherent Bessel modes can be produced, for instance, using the arrangement proposed in [6]. By choosing the modal weights given by Eq. (15), and adding up a sufficiently large number of modes, we can approximate the ideal dark (antidark) diffraction-free beam profile to good accuracy. In particular, one such fit is exhibited in Fig. 2 for dark diffraction-free beams. It is seen in the figure that the approximation is so good that the intensity profile of the ideal black diffraction-free beam, plotted in a solid curve, is practically indistinguishable from its fit with $N=25$ uncorrelated Bessel beams, displayed in a dashed curve, so long as the distance from the beam center satisfies the condition $\beta\rho<20$. In the tails of the beams, however, the two intensity profiles significantly deviate from each other. Our numerical simulations indicate that the number of modes mostly affects the behavior of the tails, not the central spot of the beam. Such a behavior persists for any N, and hence to accurately approximate a greater portion of an ideal beam, one would need a larger number of the Bessel modes. Fortunately as a dark or antidark diffraction-free beam is generated in the laboratory, the tails of the beam produced by a desired number of Bessel modes, will be "cut" by finite apertures of a source and measuring

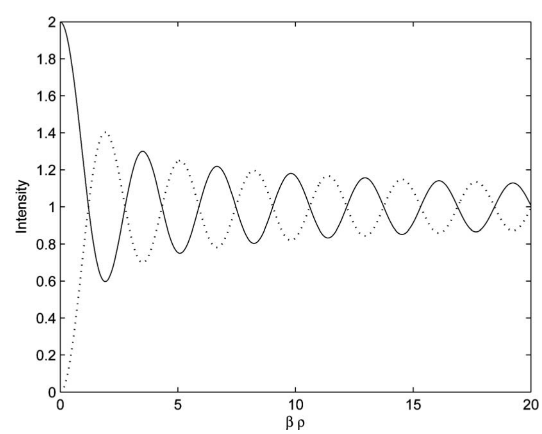

Fig. 1. Intensity profiles of dark (dashed curve) and antidark (solid curve) diffraction-free beams with $\alpha=-1$ and 1, respectively.

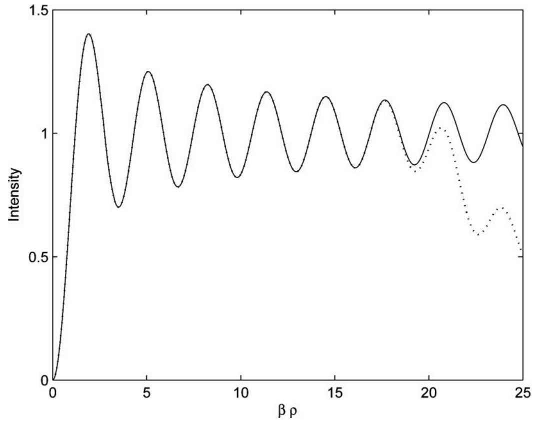

Fig. 2. Intensity profile of the ideal ($\alpha=-1$) dark diffraction-free beam (solid curve), and that of the corresponding dark diffraction-free beam, constructed with $N=25$ Bessel modes (dashed curve).

device, such as a CCD camera. The effect of the source aperture is twofold. First, the size of the aperture determines a characteristic distance over which the central bright (dark) spot of the beam remains effectively diffraction free. Second, small periodic distortions of the beam intensity profile arise due to diffraction from the aperture edges [12]. Such distortions can be removed by using the appropriate adaptive optics [19].

At this stage it is instructive to contrast the key features of novel dark (antidark) diffraction-free beams against those of the previously reported dark (antidark) solitons. One of the main differences between the two classes of nonspreading waves is their dimensionality: The discovered dark and antidark diffraction-free beams are (2+1)D waves, propagating in free space, whereas there exist only (1+1)D stable dark and antidark solitons, which require a planar waveguide for their experimental generation. Even more importantly, however, while there have been found coherent [2] as well as partially coherent dark [18] and antidark [20] solitons, the dark and antidark diffraction free beams were just shown to *always be partially spatially coherent*. Physically, it can be explained by noticing that dark or antidark beams have intensity profiles with dips or peaks, respectively, "sitting" on a cw background. It is, of course, possible to construct a fully coherent beam with such a field profile by considering a *coherent* superposition of a (reference) plane wave of the amplitude U_∞ with a diffraction-free beam, $U(\boldsymbol{\rho},z)\propto U_\infty+U_{df}(\boldsymbol{\rho})e^{i\gamma z}$. However, the combined field will not be diffraction free due to the presence of the interference (beat) term in the expression for the intensity profile $I(\boldsymbol{\rho},z)\equiv|U(\boldsymbol{\rho},z)|^2$. In the partially coherent case, all Bessel modes are superposed *incoherently* such that no interference terms are present, and as a result, a diffraction-free dark or antidark beam is formed.

Finally, we mention that the present dark diffraction-free beams can serve as atomic traps. The atoms can be trapped in the vicinity of a dark notch of the beam where the optical field vanishes.

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